Démontrons par récurrence que $\forall n \in N;\quad Z_n = a^n Z_0$

$Z_1 = aZ_0$

Supposons que $Z_n = a^n Z_0$ et montrons que : $Z_{n+1} = a^{n+1} Z_0$

On a : $Z_{n+1} = aZ_n = a(a^n Z_0) = a^{n+1} Z_0$

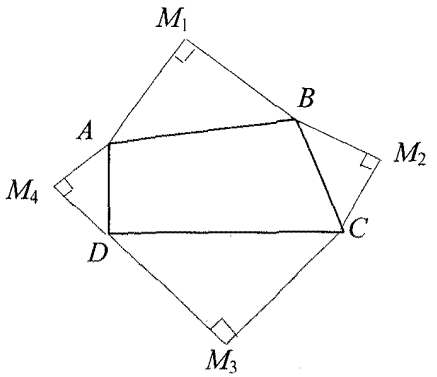
donc $\forall n \in \mathbf{N};\quad Z_n = -e^{i\frac{2\pi}{3}n}(z_0 - e^{i\frac{\pi}{6}})$

or $z_0 = 0 \Rightarrow Z_n = e^{i(\frac{2\pi}{3}n+\frac{\pi}{6})}$ donc $\forall n \in \mathbf{N}:\quad Z_n = e^{i(\frac{2\pi}{3}n+\frac{\pi}{6})}$

On a : $z_n = Z_n + e^{i\frac{\pi}{6}} = e^{i(\frac{2\pi}{3}n+\frac{\pi}{6})} + e^{i\frac{\pi}{6}} = e^{i\frac{\pi}{6}}(e^{i\frac{2\pi}{3}n} + 1)$

$$z_{2000} = \frac{\sqrt{5}}{2} - \frac{1}{2}$$

Solution 19 :



1°/ Le triangle M_1AB est isocèle rectangle en M_1 de sens

$$\text{direct.} \Leftrightarrow \begin{cases} M_1A = M_1B \\ (\overrightarrow{M_1A}, \overrightarrow{M_1B}) \equiv \dfrac{\pi}{2}(2\pi) \end{cases} \Leftrightarrow \begin{cases} \dfrac{M_1B}{M_1A} = 1 \\ (\overrightarrow{M_1A}, \overrightarrow{M_1B}) \equiv \dfrac{\pi}{2}(2\pi) \end{cases}$$